Week 5
Introduction to Monte Carlo Methods

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Introduction :

What we will learn today :

Introduction to Monte Carlo Methods

Estimator for integration

Introduction

Monte Carlo estimation is based on the relative frequency interpretation of probability.

Random experiment: outcome of performing the experiment, cannot be predicted exactly but can be defined statistically

Executing the random experiment a large number of times, *N*.
If the number of occurrences N_A of an event A.

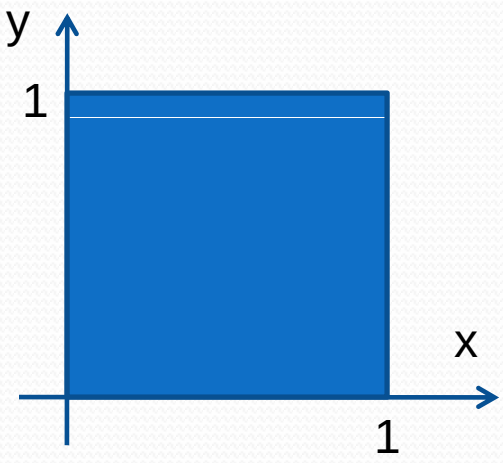
The probability of the event A is approximated by the relative frequency of the event, which is defined by

$$P(A) = \lim_{N \to \infty} \frac{N_A}{N}$$

Monte Carlo Estimation

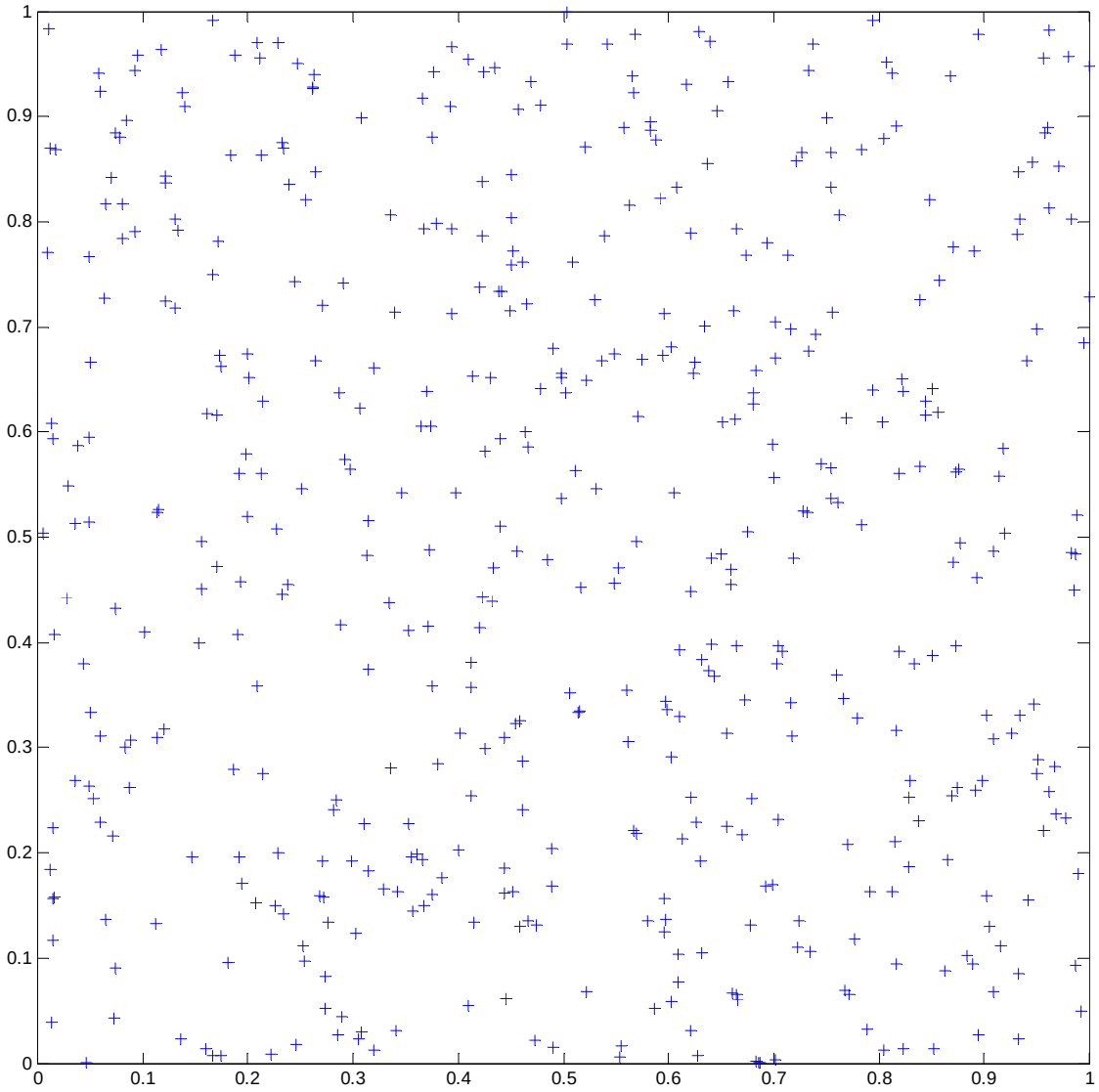
Basic example for establishing understanding:

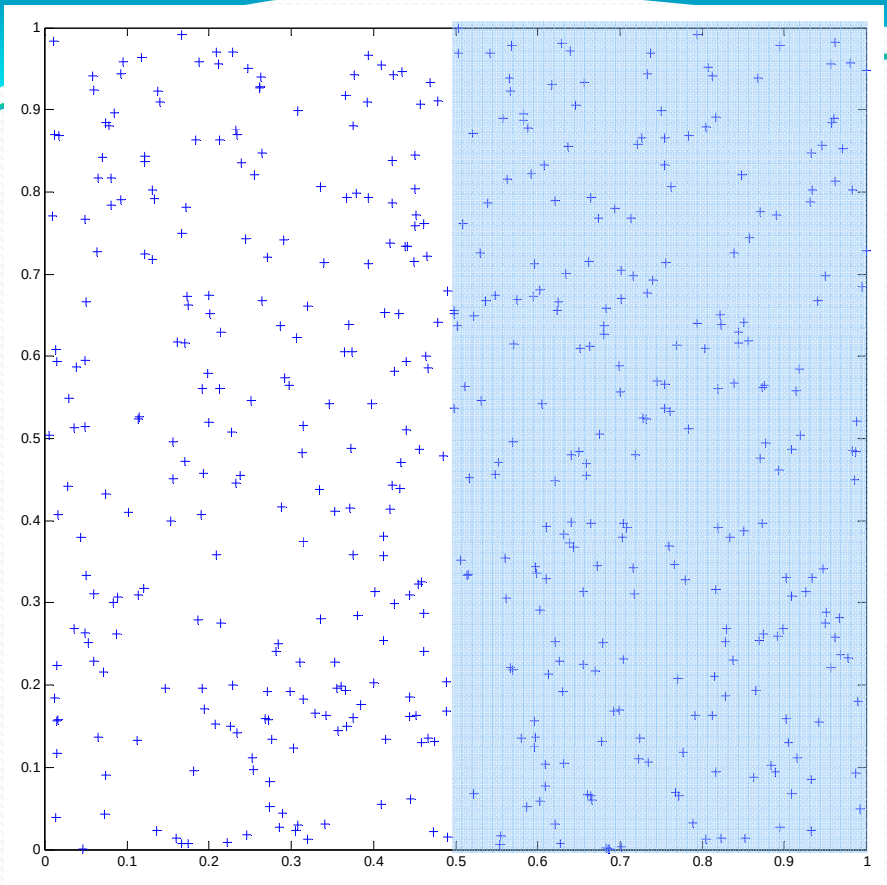
Generating N coordinate points (a pair of numbers) x and y, that are uniformly distributed in the are (0,0) and (1,1), or the area as indicated below



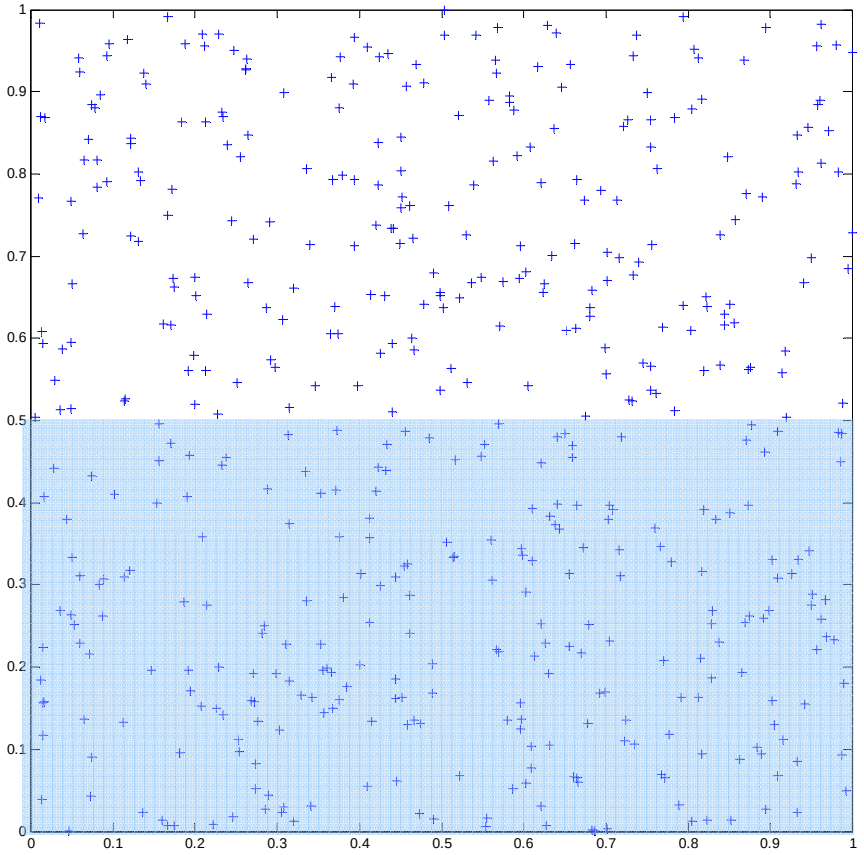

```
P=rand      % generate uniformly distributed random number
            % between 0 and 1

P=rand(N,2)   %  generate N pairs uniformly distributed
              %  random  numbers

x= P(:,1);    % correspond the first column to x
y= P(:,2);    % correspond the second column to y

plot(x,y,'+'); axis equal;axis([0 1 0 1]);
            % draw the points
```

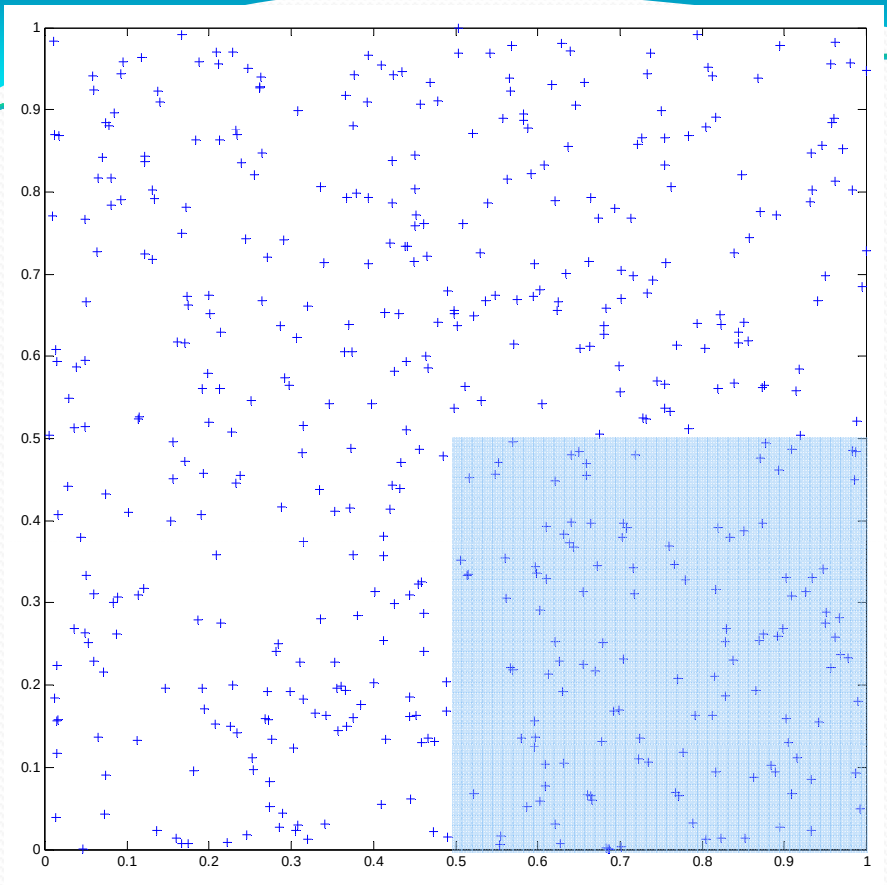
For N=500;
P=rand(500,2) %generate 500 points uniformly distributed



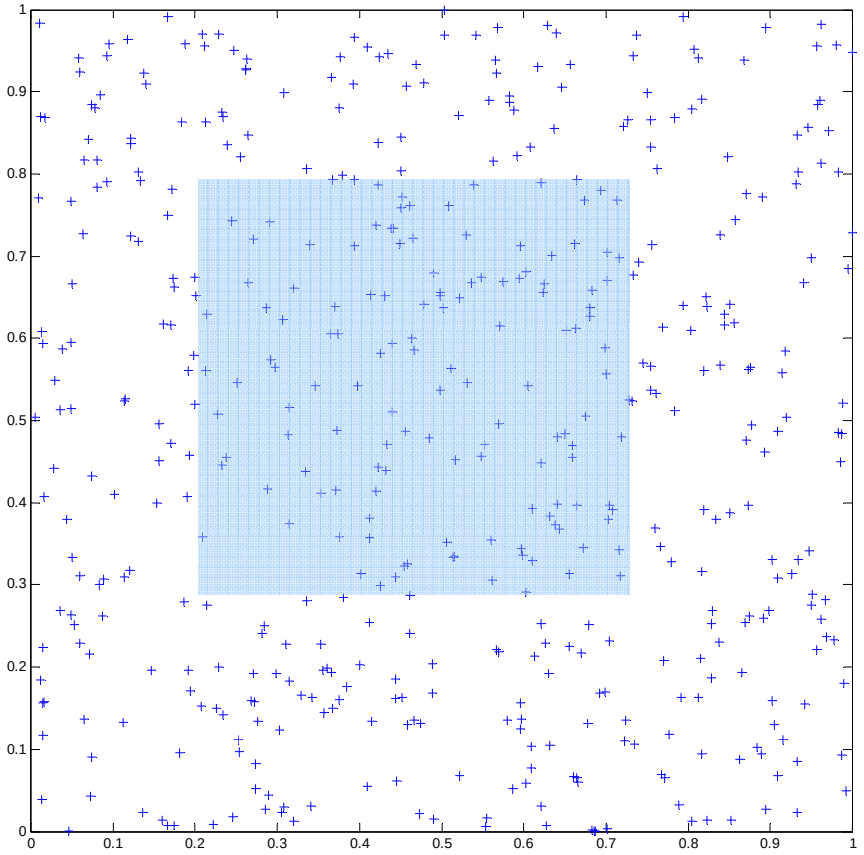


For N=500,
How many '+' are located in the shadowed area?



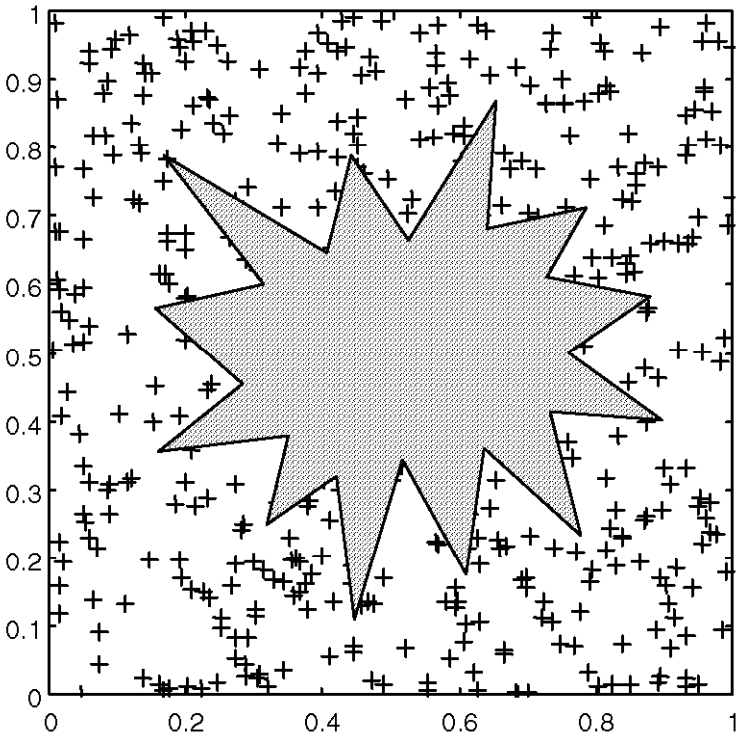


For N=500,
How many ‘+’ are located in the shadowed area?



The area of shadowed region is proportional to the number of '+'.

$$\frac{N_A}{N} \approx \frac{A}{A_{box}}$$



What is the area of the sunburst ?

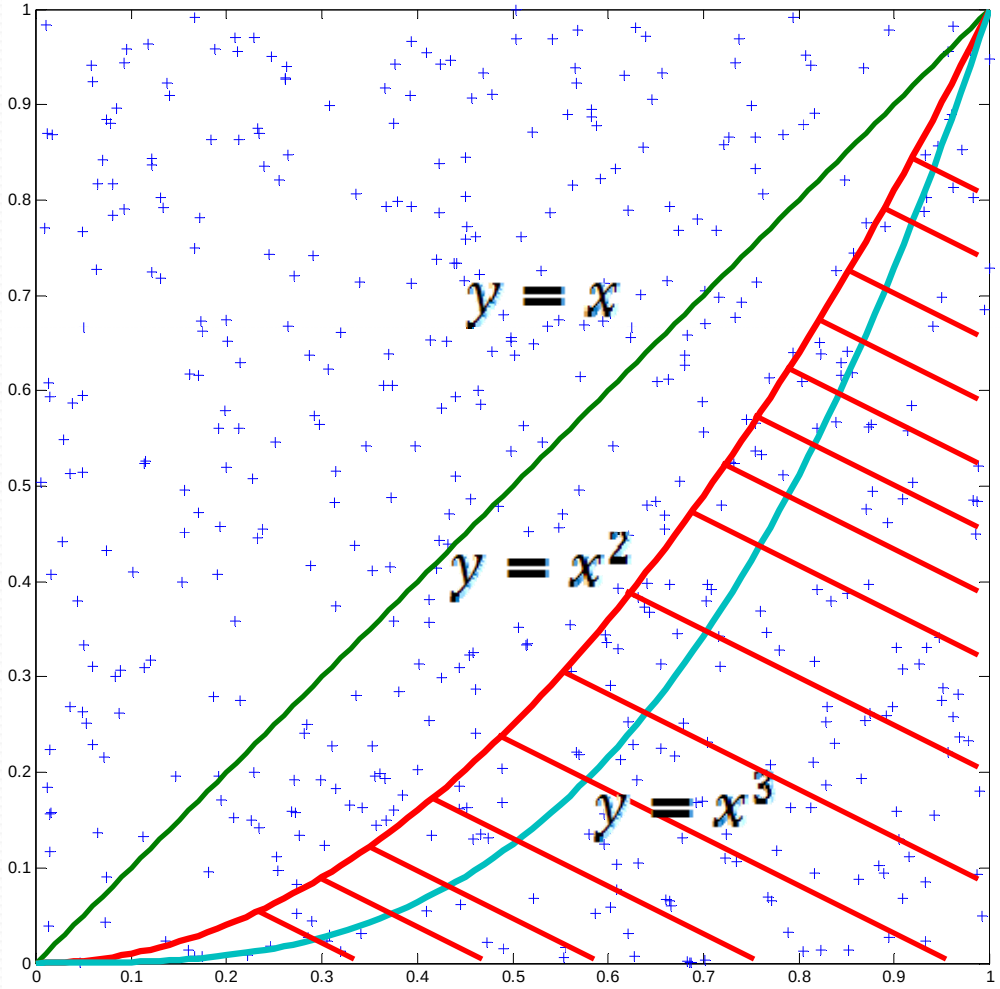
$$A \approx \frac{N_A}{N} A_{box}$$

N_A Number of '+' inside sunburst

N Total number of '+'

A_{box} (in this case 1)

How can we calculate the area under the curve y=x*x (red area) ?



Analytically we can get it by integration:

$$\int_0^1 x^2 dx = \frac{1}{3}[x^3]_0^1 = \frac{1}{3}$$

How can we check that the '+' is located in the red area ?

Mathematically, the red area can be described as:

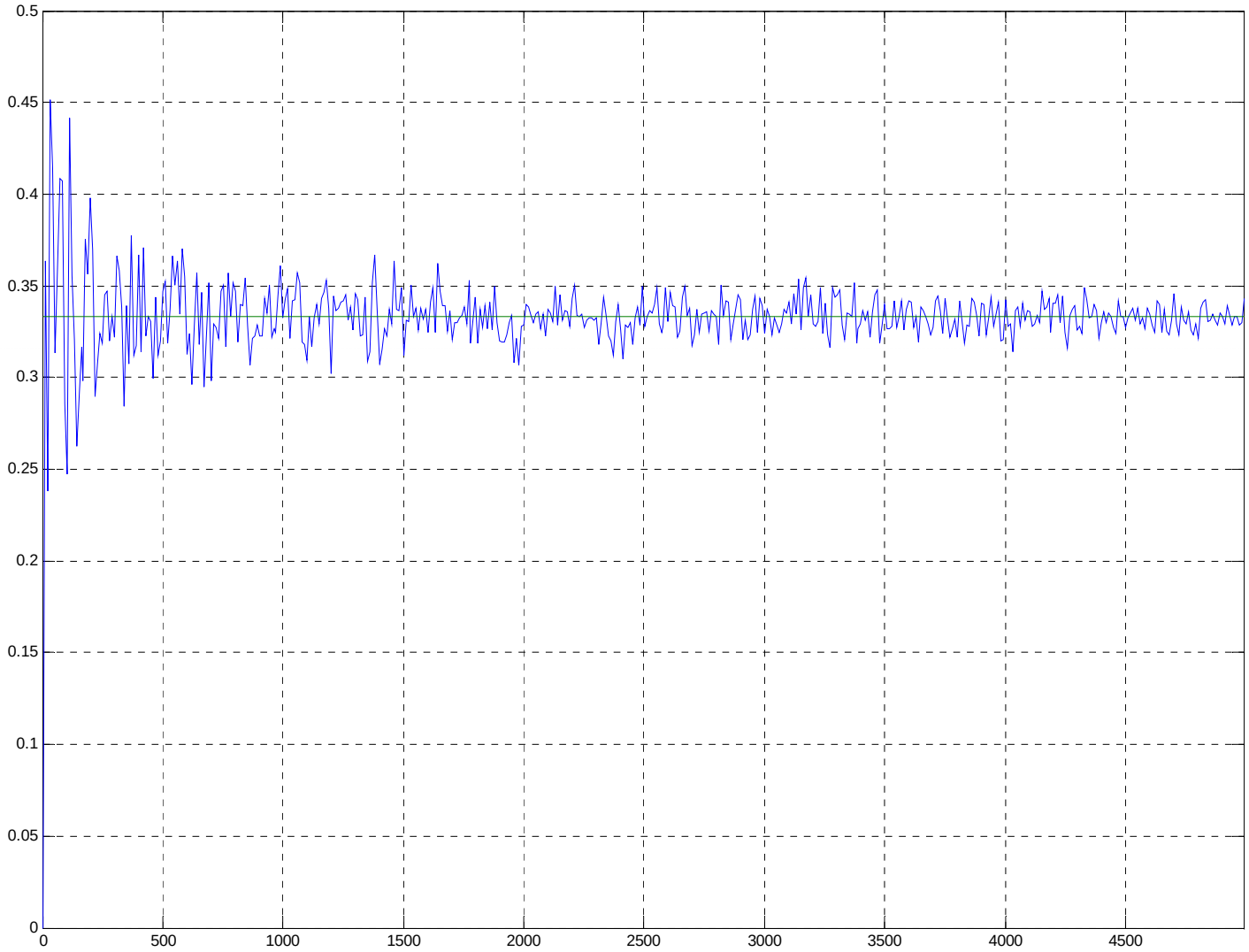
$$x^2 \geq y$$

```
n=500;
P=rand(n,2);
x=P(:,1);
y=P(:,2);
k=0;
for i=1:n
   if  x(i)*x(i)>=y(i)       % Fall inside red area?
    k=k+1;
  end
end
nilai=k/n
```

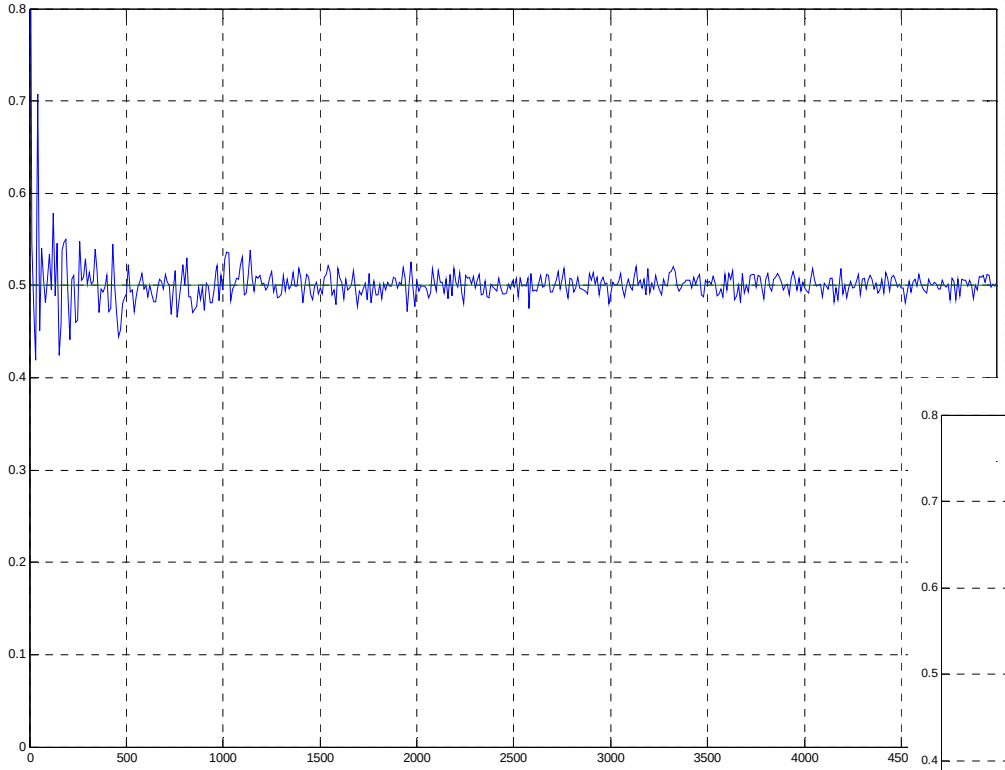
If we start this program multiply, we get for example following results”

0.3300
0.3060
0.3320
0.3440
etc

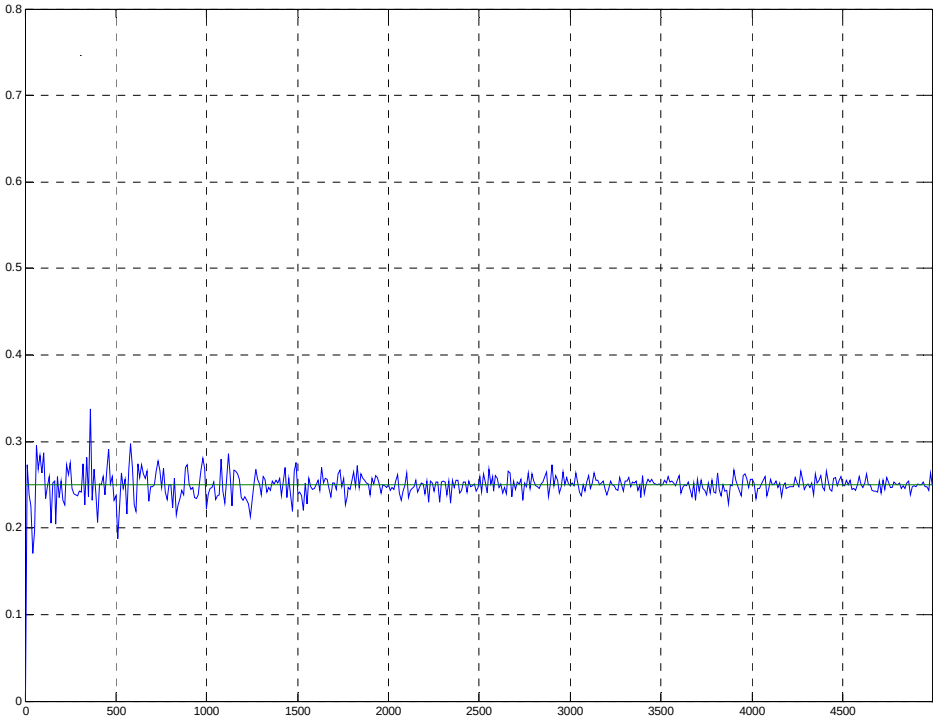
For large value of N, the results close to exact solution (1/3)



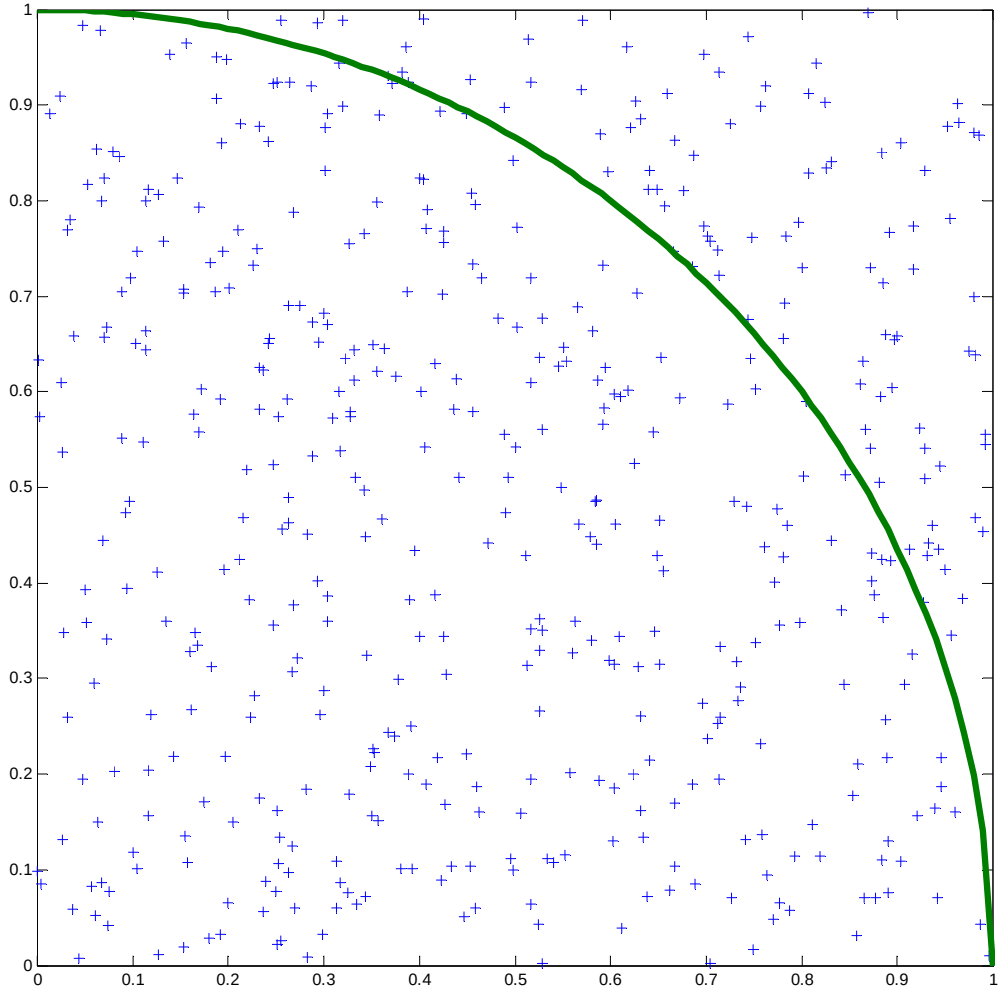
for $y = x$



for $y = x^3$



Estimating π



The area of a quarter of circle is π/4

```
k=0;
for i=1:n
 if  sqrt(1-x(i).*x(i))>=y(i)
    k=k+1;
 end
end
nilai=4*k/n;
```