# Week 6
# Monte Carlo Methods

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## What we will learn today :

More on integrations

Buffon's needle experiment

Problem: solving Sudoku using Monte Carlo

Do in class:

1. The exact result of the following integration is given

$$\int_{0.5}^{1} \cos x \, dx = 0.3620$$

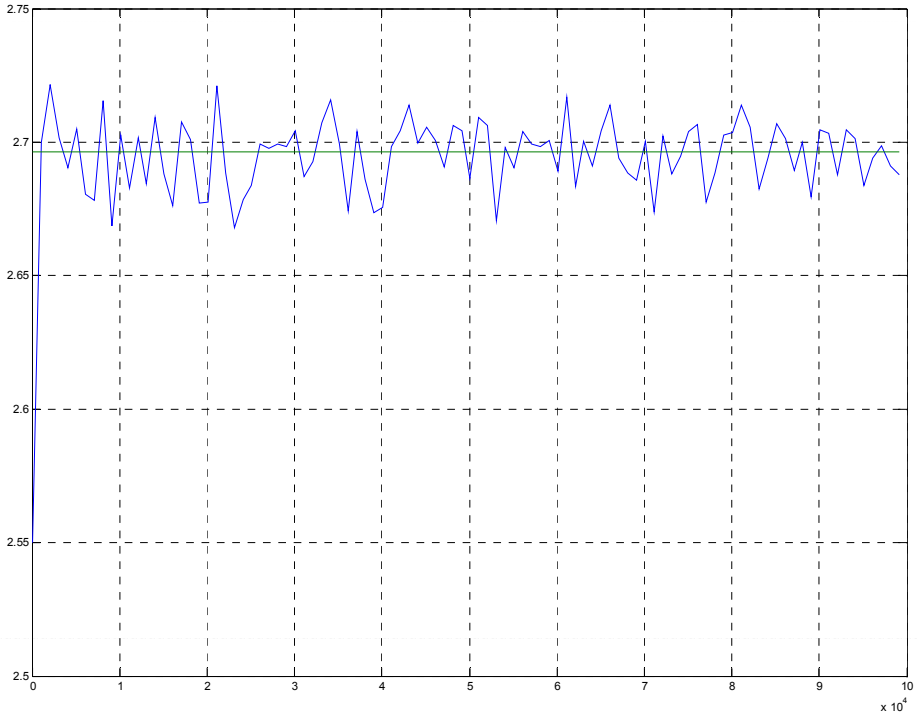
Approximate with Monte Carlo method

2. The exact result of the following integration is given

$$\int_{1.5}^{4} 4e^{-\frac{x}{2}} \, dx = 2.6963$$

Approximate with Monte Carlo method


0.37
0.365
0.36
0.355
0
1
2
3
4
5
6
7
8
9
10
x 10^4

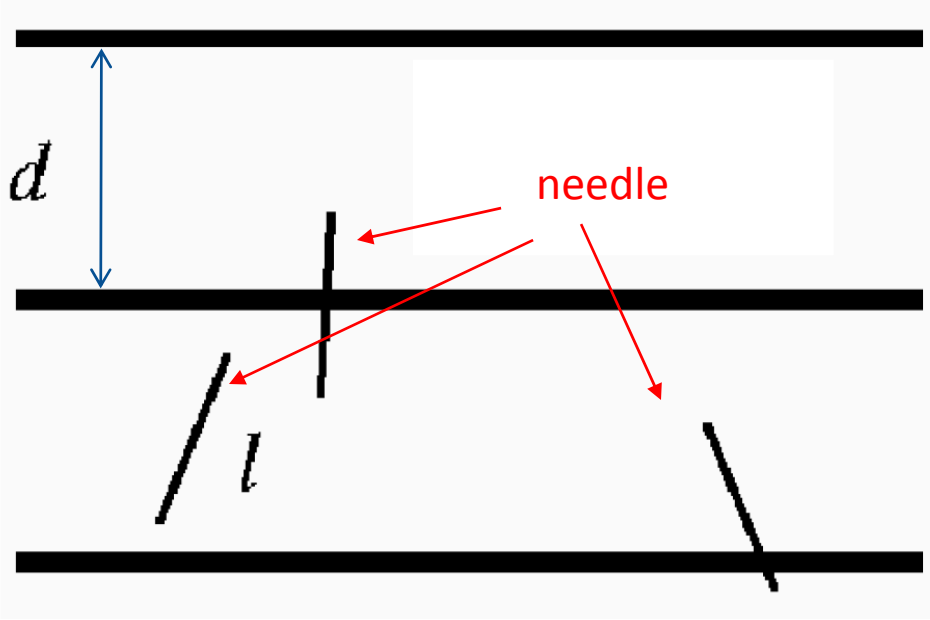

2.75
2.7
2.65
2.6
2.55
2.5
0
1
2
3
4
5
6
7
8
9
10
x 10^4

## Buffon's Needle

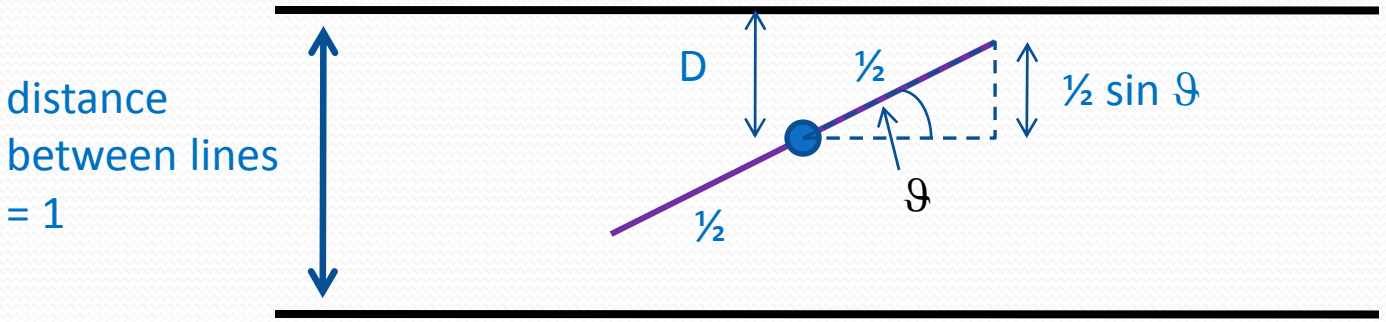
is one of the oldest problems in the field of geometrical probability (1777)

Dropping a needle on a lined sheet of paper and determining the probability of the needle crossing one of the lines on the page.

The remarkable result is that the probability is directly related to the value of pi.

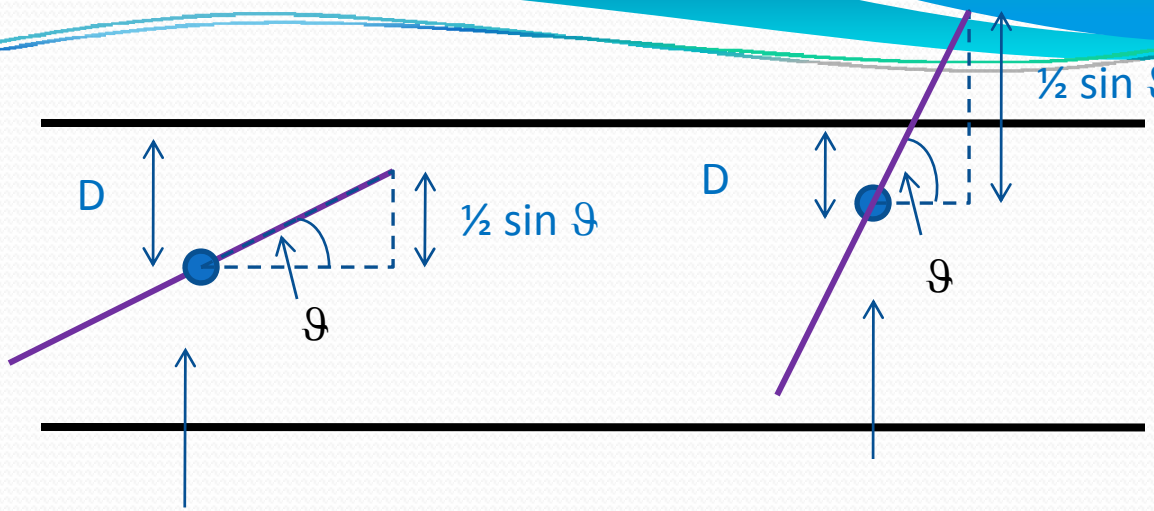


Simple case first : the length of the needle is one unit and
the distance between the lines is also one unit.




Two random variables : the angle at which the needle falls /ϑ and
the distance from the center of the needle to the closest line/D

ϑ varies from 0 to 180 degrees and D ≤ ½




miss the line

hit the line

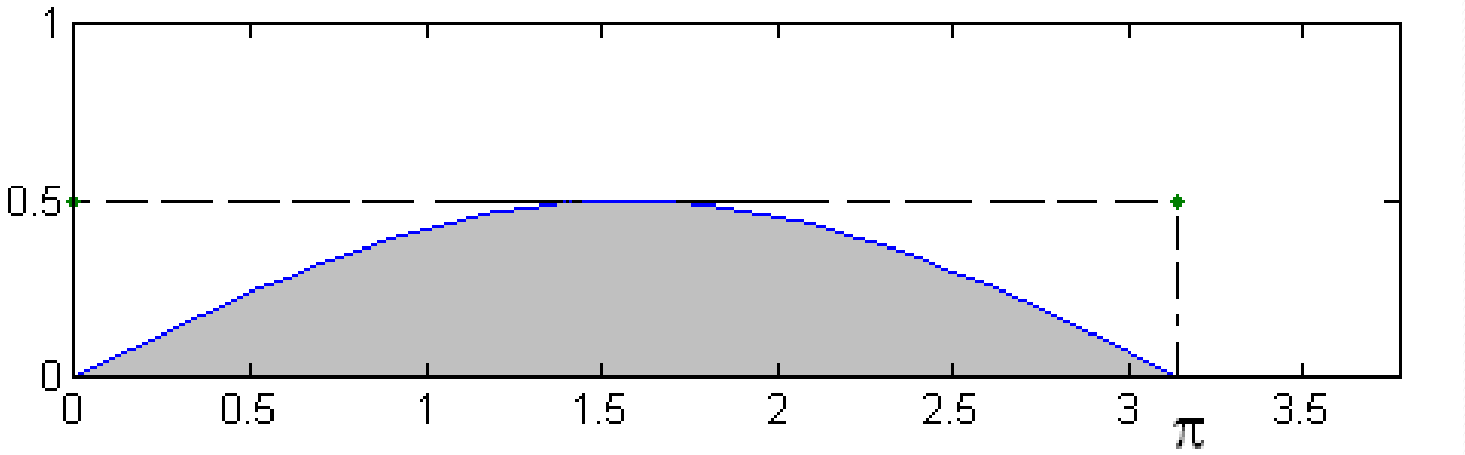
The left needle in the picture misses the line.
The right needle hits the line.

The needle will hit the line if the closest distance to a line (D) is less than or equal to 1/2 times the sine of theta.

$$D \leq \tfrac{1}{2} \sin \vartheta$$

How often will this occur?

The function D ≤ ½ sin ϑ is the shaded region in the picture below.

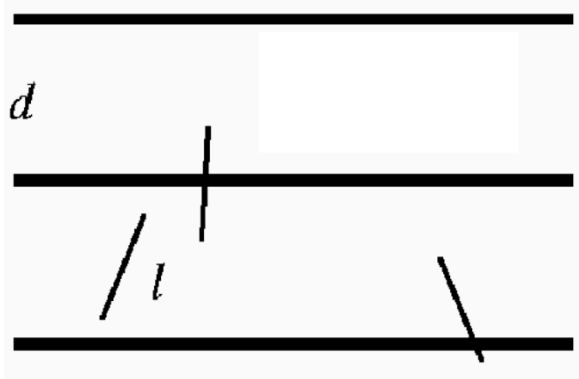



The area of the shaded region is 1.
The total area is ½ π.

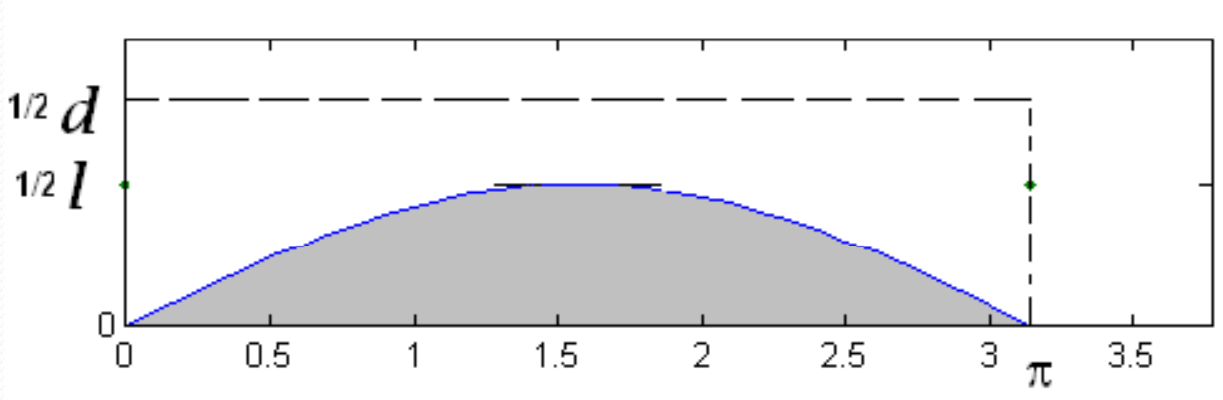
If the number of hit $N_{hit}$ and the total throw is N, we get the relationship

$$\frac{N_{hit}}{N} \approx \frac{1}{\frac{1}{2}\pi} \quad \Rightarrow \quad \pi \approx \frac{2N}{N_{hit}}$$

For general problem:




length of needle *l*
and
line distance *d*




The shaded area : ½ *l*
and
the total area: ½ *d*

$$\pi \approx \frac{2N}{N_{hit}} \frac{l}{d}$$

## The code :

Two random data

position of the middle point of needle to the line

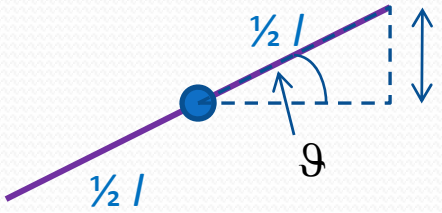
randomized between 0 and ½ d

```
D=0.5*d*rand(N,1);     % N number of throw
```

orientation of the needle in respect to the horizont

```
theta=pi*rand(N,1);    % unfortunately we use here pi (smile)
```

Calculating the front side of the triangle



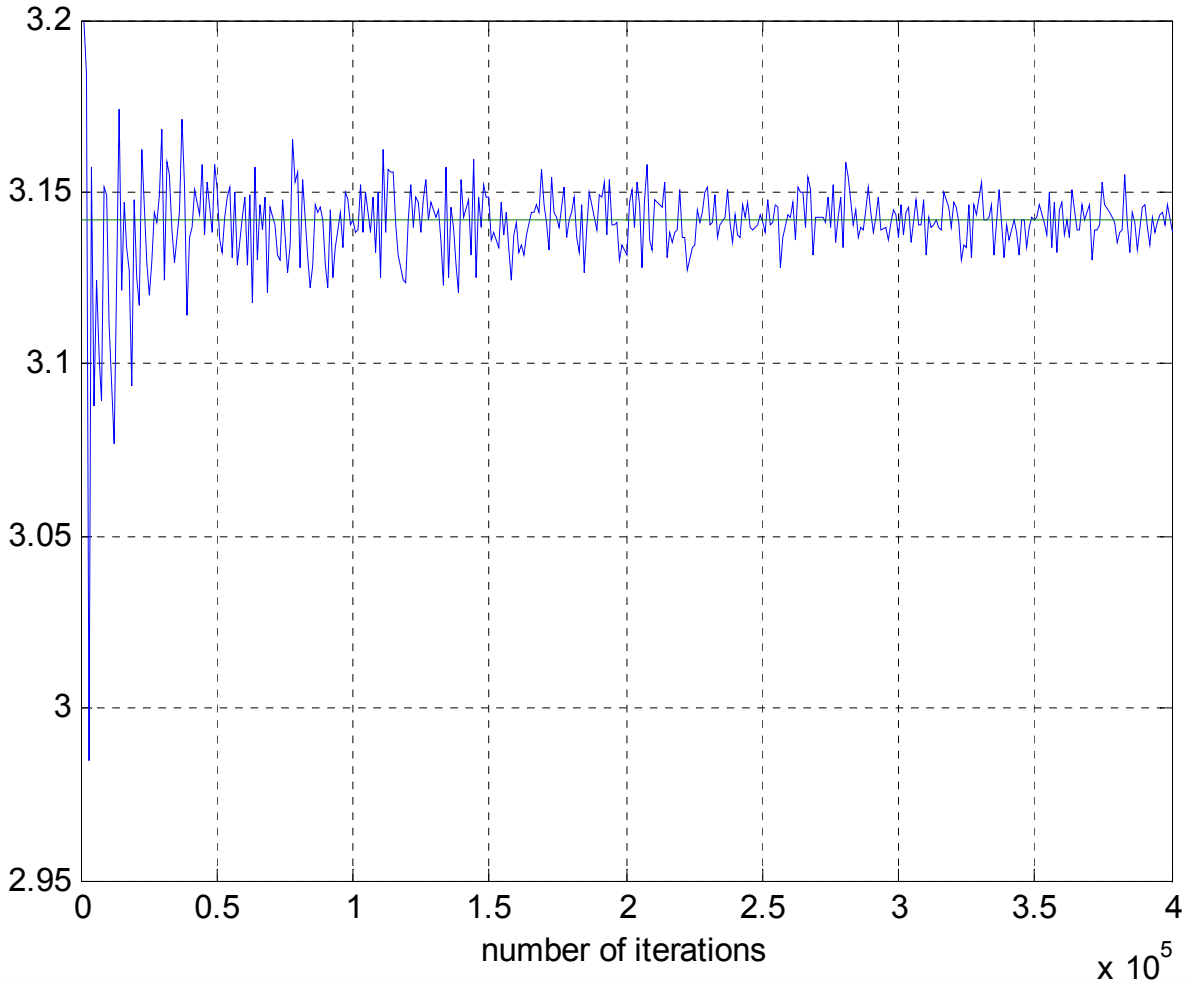

```
sisi_depan=0.5*L*sin(theta);
```

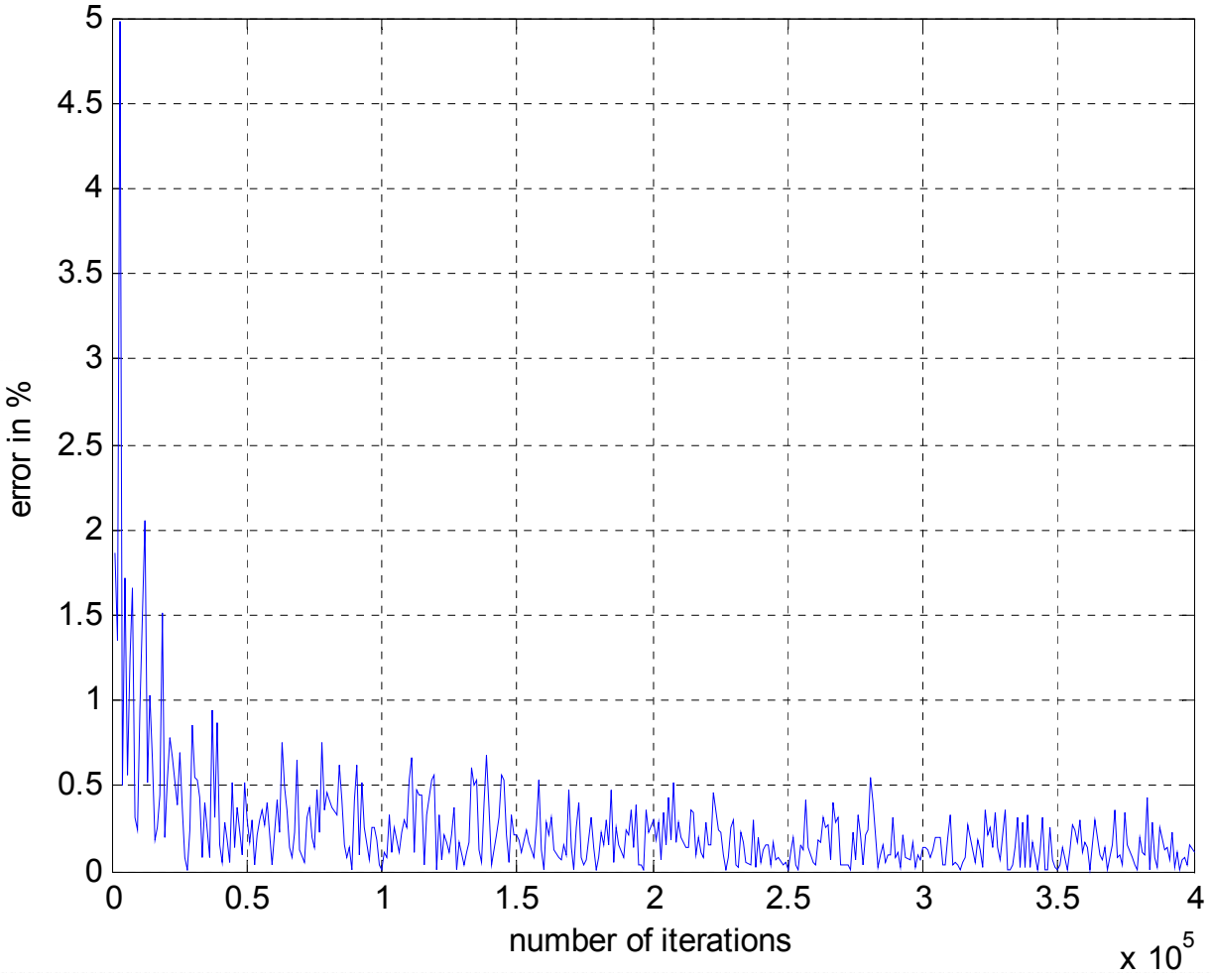
Checking hit of miss : hit if D ≤ ½ *l* sin ϑ

```
if D(ii)<=sisi_depan(ii)
        hit=hit+1;
end
```

The complete code:

```
d=1;              L=0.8;
n=1000:1000:400000;
aa=length(n);
 for j=1:aa,
    N=n(j);
    D=0.5*d*rand(N,1);
    theta=pi*rand(N,1);
    sisi_depan=0.5*L*sin(theta);
    hit=0;
    for ii=1:N,
      if D(ii)<=sisi_depan(ii)
         hit=hit+1;
      end
     end
   P=hit/N;
   pi_aprox(j)=2*L/d/P;
end
Plot(n,pi_aprox,[0   max(n)],[pi    pi]);grid
```


3.2
3.15
3.1
3.05
3
2.95
0
0.5
1
1.5
2
2.5
3
3.5
4
number of iterations
x 10^5


error in %
number of iterations
x 10^5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

# Sudoku

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | | | 7 | | | | |
| 6 | | | 1 | 9 | 5 | | | |
| | 9 | 8 | | | | | 6 | |
| 8 | | | | 6 | | | | 3 |
| 4 | | | 8 | | 3 | | | 1 |
| 7 | | | | 2 | | | | 6 |
| | 6 | | | | | 2 | 8 | |
| | | | 4 | 1 | 9 | | | 5 |
| | | | | 8 | | | 7 | 9 |

Solving with Monte Carlo method ??

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 4 | 6 | 7 | 8 | 9 | 1 | 2 |
| 6 | 7 | 2 | 1 | 9 | 5 | 3 | 4 | 8 |
| 1 | 9 | 8 | 3 | 4 | 2 | 5 | 6 | 7 |
| 8 | 5 | 9 | 7 | 6 | 1 | 4 | 2 | 3 |
| 4 | 2 | 6 | 8 | 5 | 3 | 7 | 9 | 1 |
| 7 | 1 | 3 | 9 | 2 | 4 | 8 | 5 | 6 |
| 9 | 6 | 1 | 5 | 3 | 7 | 2 | 8 | 4 |
| 2 | 8 | 7 | 4 | 1 | 9 | 6 | 3 | 5 |
| 3 | 4 | 5 | 2 | 8 | 6 | 1 | 7 | 9 |