# Study of SIR for Designing Filters with Arbitrary Resonant Positions

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***Abstract*—Stepped impedance resonators are a structure consisting of several transmission lines with different admittances. They are used for control the spurious response and insertion loss of filters by changing the impedance/admittance ratio of the stepped impedances. The multiple mode resonator, which is implemented in form of stepped impedance resonator, can be used, either to widen pass/stop bands or to separate pass bands and stop bands from each other. In this work, we study a stepped impedance structure consisting of two transmission line segments, which have the general form of arbitrary ratio between the admittances and arbitrary ratio of lengths. We observe the appearance and movement of the resonances and verify them through computer simulation in microstrip technology. Variation of these ratios lead to appearance and moving of resonant positions. This approach was numerically verified by means of computer simulation with a commercial software. The accuracy of the predicted resonant positions is very good, i.e. about 48 MHz at 4.6 GHz and about 76 MHz at 6.8 GHz.**

***Keywords*—*filter, microstrip, multiple-mode, resonator, stepped impedance resonator***

## I. Introduction

The recent advanced digital communications systems require efficient use of the frequency spectrum. The use of this scarce expensive resource must be controlled strictly otherwise significant interference can occur, which can deteriorate the overall system performances. Bandpass filters are designed for this purpose. Bandpass filters pass desired signals from other unwanted signals. To get sharp filtering characteristics, in traditional filter approximations, such as Butterworth and Chebychev realizations, we need many resonators, which leads to big filter dimensions. The introduction of transmission zeros enhances the selectivity near the pass band [1], [2].

Transmission line structures consist of stepped impedances, i.e. two transmission segments with different values/width connected directly to each other, are used often as resonators. In [3] stepped impedance resonators (SIRs) are used for control the spurious response and insertion loss of filters by changing the impedance/admittance ratio of the stepped impedances. A further study [4] standardized SIR in the length combinations $\lambda_g/4$, $\lambda_g/2$ and $\lambda_g$ and summarized systematically their fundamental characteristics, such as resonance conditions, resonator length, spurious (higher order) responses, and equivalent circuits. Application of SIR for WLAN was introduced in [5]. The filter has a dual-band feature at 2.45 and 5.75 GHz and a 85 dB suppression at 3.5 GHz.

A tri-section SIR was reported in [6], the structure features a short and low-impedance section inserted in a two-section SIR. The inserted section makes the resonator more compact, and enables the flexibility of introducing cross coupling in a filter configuration. Using the tri-section SIR, a cascaded triplet bandpass filter is demonstrated, p sharp roll-off at the high edge of the passband.

In [7] a microstrip-line ultra-wideband (UWB) bandpass filter is proposed and implemented using a multiple-mode resonator (MMR) which is a cascaded combination of SIRs. The filter is designed for the whole UWB passband of 3.110.6 GHz. In the design, the first three resonant frequencies of this MMR are properly adjusted to be placed quasiequally within the UWB. Then, the parallel-coupled lines at the two sides are longitudinally stretched so as to raise the frequency-dispersive coupling degree with the coupling peak near the center of the UWB. After optimization of this filter, a good UWB bandpass behavior with five transmission poles is theoretically realized and experimentally confirmed. Within the whole UWB passband, the return loss is found higher than 10 dB, and the group delay variation is less than 0.23 ns.

The multiple mode resonator (MMR), which is implemented in form of stepped impedance resonator (SIR), can be used, either to widen pass-/stopbands or to separate passbands and stopbands from each other. In this work, we study a stepped impedance structure consisting of two transmission line segments, which have the general form of arbitrary ratio between the admittances and arbitrary ratio of lengths. We observe the appearance and movement of the resonances and verify them through computer simulation in microstrip technology. As simulation tool, the software Sonnet v.13 is used [8].

## II. Stepped Impedance Resonator

The simplest form of a stepped impedance resonator (SIR) as introduced in [3] is depicted in Fig. 1. The SIR consists of two transmission lines with different characteristic admittances $Y_1$ and $Y_2$ and in general form also it has different lengths $l_1$ and $l_2$.
In this work, we observe only the open condition at the position $LL$', so that the input admittance of the lossless transmission line at the position $AA$' seen to the right becomes

$$Y_{in}^{AA'} = jY_1 \frac{Y_2 \tan\theta_2 + Y_1 \tan\theta_1}{Y_1 - Y_2 \tan\theta_2 \tan\theta_1} \quad (1)$$

$\theta_1$ and $\theta_2$ are the phases in the first and second transmission lines, respectively.

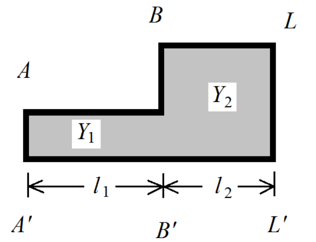


Fig. 1. Stepped Impedance Resonator (SIR) consists of two microstrip lines with characteristic admittances of $Y_1$ and $Y_2$ and lengths $l_1$ and $l_2$.

Due to open condition at the load and lossless transmission lines, the input admittance in Eq. (1) is imaginary. At resonances, the imaginary part of the admittance must be zero.

In this work, we define the ratio between the characteristic admittance $\eta = Y_2/Y_1$ and the ratio between the lengths $\nu = l_2/l_1$. The length of the transmission lines together with the appropriate phase constants leads to the phase as arguments of each tangent functions in Eq. 1. Due to the nature of the phase constant as nonlinear function of the geometrical and material data of the transmission line, we set approximately, with $\theta_1 = \theta$, $\theta_2 = \nu\theta$. In this way, by given $\eta$ and $\nu$, the value of the input admittance changes with $\theta$. In this way Eq. 1 becomes

$$\frac{Y_{in}^{AA'}}{jY_1} = \frac{\eta \tan \nu\theta_1 + \tan \theta_1}{1 - \eta \tan \nu\theta_1 \tan \theta_1} \tag{2}$$

If the ratio of the admittances $\eta$ and ratio of the length $\nu$ are given, the resonances can be found by locating the angles $\theta_1$ by some root finding methods as described in section III.

## III. Root Finding

At given values of $\eta$ and $\nu$ as described in previous section, resonant conditions happen if the input admittance as formulated in Eq. 2 becomes zero. This happen at certain frequencies. Fig. 2 shows an example for the case $\eta = 2$ and $\nu = 2$. The graph is depicted for $0 < \theta_1 < 3.46$.

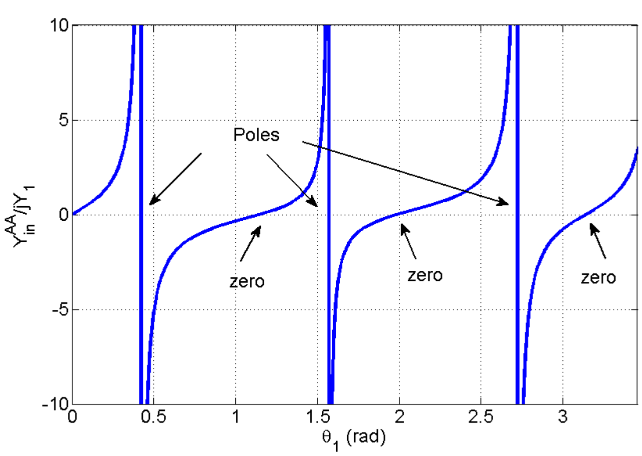


Fig. 2. The input admittances seen at AA' as function of $\theta_1$ with admittance ratio $\eta = 2$ and length ratio $\nu = 2$.

We see in this observation interval three zeros and three poles. The first zero is the resonant frequency of the fundamental mode. The second and so on are the resonant frequencies of higher modes. If we include several modes in our circuit, it means, the circuit works with multiple modes. In order to locate zeros from the graph, a root finding method can be used [9]. A powerful and simple method is bracketing and bisection method. In its original form, bracketing and bisection method recognizes poles also as zeros, however with additional constraint we can distinguish them and deliver the correct zeros as output of this root finding method.

The bisection method is slow, but for our problems here it is fast enough. We need less than 1 second time to find all correct zeros in interval $0 < \theta_1 < 13.2$ with accuracy of 0.001 executed in an i3-computer with 4GB RAM.

## IV. Results

In this work, firstly we study the variation of ratio of the admittances and the ratio of the lengths to the position of the resonant points. At the end, we verify the study in microstrip technology.

### A. Parameter Study

For the special case that $\nu = 1$, the tangent functions in the denominator possess the same argument. It is simple to see, that the zeros of the input admittance is equal the zeros of $\tan \theta_1$, independent on the ratio of the admittances $\eta$. Fig. 3 shows this phenomenon.

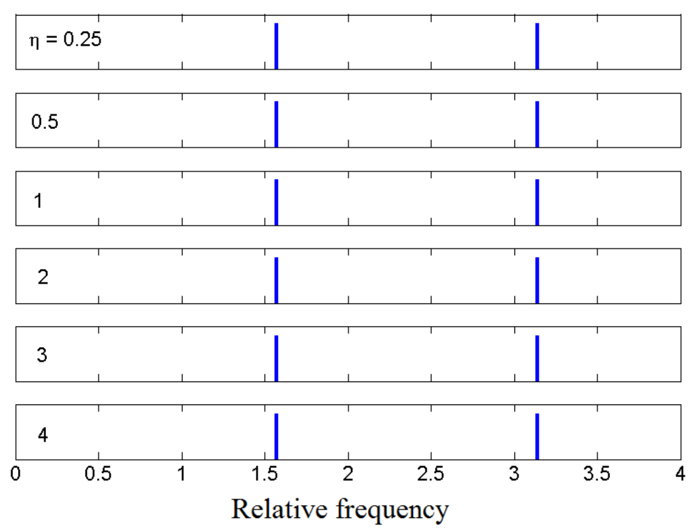


Fig. 3. The resonant frequencies for length ratio $\nu = 1$ with different admittance ratio $\eta$.

However, it is worthy to note, that for different admittances, a microstrip line has different strip widths which yield to different phase constants. This implies that the physical length of the real circuit can have different dimensions. So that, if you change the $\eta$ without changing the physical length of the line, the resonant positions can shift slightly.

The condition is different, if the lengths of the transmission lines are different $\nu \neq 1$. We see by increasing the ratio $\nu$ from

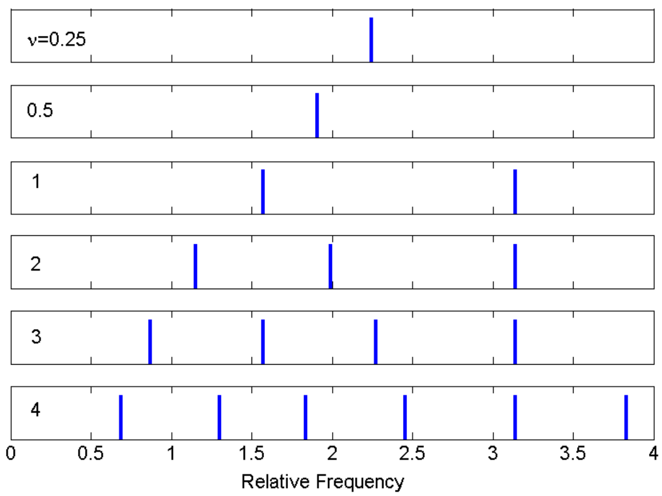


Fig. 4. The resonant frequencies for admittance ratio $\eta = 2$ with different length ratio $\nu$.

0.25 to 4, the number of resonances in the interval $0 < \theta_1 < 4$ increases. Fig. 4 and 5 show the resonant positions for cases $\eta = 2$ and $\eta = 0.5$, respectively.

Due to larger dimension, the first resonant position is shifted to a lower frequency value. Furthermore, we see, that independent on $\eta$, for whole number of $\nu$, i.e. $\nu = 1, 2, 3$, etc, due to zeros of $\tan\theta_1$ and $\tan\nu\theta_1$,we have resonances at 3.1416.

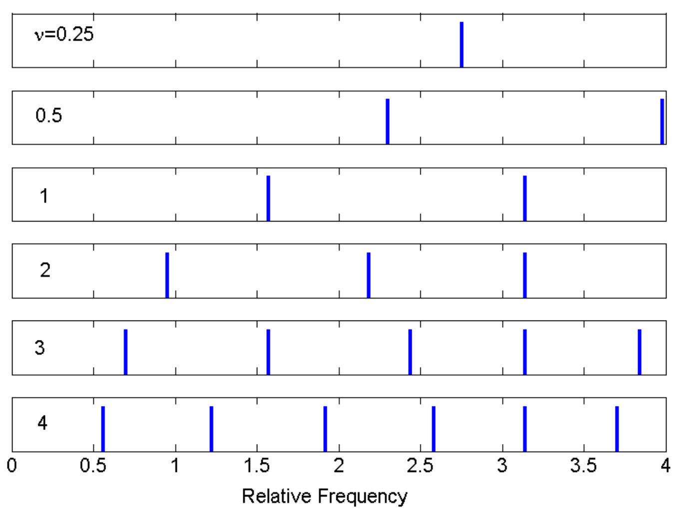


Fig. 5. The resonant frequencies for admittance ratio $\eta = 0.5$ with different length ratio $\nu$.

## B. *Simulation of SIR in Microstrip Technology*

To verify the results obtained in previous section, here we implement the resonator in microstrip technology for the first resonant frequency at 2.3 GHz. As material, we use a substrate based on Rogers TMM10 ($\varepsilon_r = 9.56$ and $\tan\delta = 0.0022$) with thickness $h = 0.635$ mm. At first, we take the case $\eta = 4$ and $\nu = 1$. The zeros found by root finding method are 1.5705, 3.1415 and 4.7125. As next, we must choose $Y_1$ or $Z_1 = 1/Y_1$, which leads neiter too thin nor too thick strip. We chosen $Z_1 = 86.5\,\Omega$ and with Tx Line Calculator [10] we get the strip width $w_1 = 0.15$ mm. At the frequency 2.3 GHz, the phase constant becomes $\beta_1 = 0.117614$ rad/mm, so that to aim the first resonant (1.5705 rad) at this frequency, we need a transmission line with the length $l_1 = 1.5705/0.117614$ mm $= 13.35$ mm. The second microstrip line has the impedance of $Z_2 = Z_1/\eta = 21.625\,\Omega$, which means it has the strip width $w_2 = 2.45$ mm. At the frequency 2.3 GHz, the second microstrip line has the phase constant $\beta_2 = 0.132017$ rad/mm, and the physical length $l_2 = 11.9$ mm. Fig. 6 shows the microstrip circuit designed in software Sonnet v.13 [8]

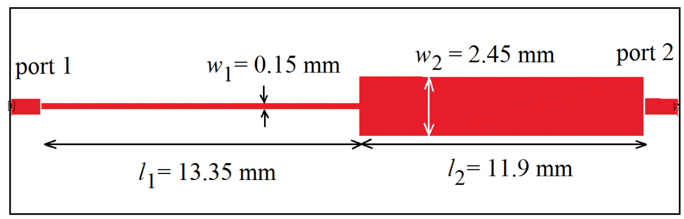


Fig. 6. Microstrip implementation of a stepped impedance resonator in TMM10 (thickness 0.635 mm) with $\eta = 4$ and $\nu = 1$ for resonant frequency 2.3 GHz.

The SIR is coupled weakly to the input and output ports. The first resonant frequency is set to 2.3 GHz. The second resonance happens at phase 3.1415 rad, which for the geometry means at 4.588 GHz and the third at 6.856 GHz as indicated with red circles in Fig. 7.

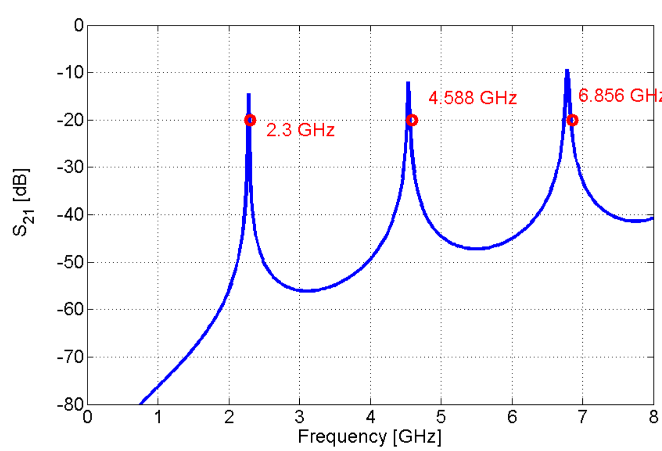


Fig. 7. The transmission factor $S_{21}$ calculated by Sonnet (solid line) and the resonant frequencies predicted by analytical approach (o in red), for case $\eta = 4$ and $\nu = 1$.

By Comparing these results with the result obtained by Sonnet (solid line), which delivers the transmission factor with resonant positions at 2.3 GHz, 4.54 GHz (48 MHz shifted) and 6.78 GHz (76 MHz shifted compared by analytical result), we have a very accurate analytical approach.

In Fig. 8 we verify, that independent of the value $\eta$ for $\nu = 1$ the positions of resonant frequencies do not change. However, because of different strip widths, there are different phase constants, which must be compensated with different

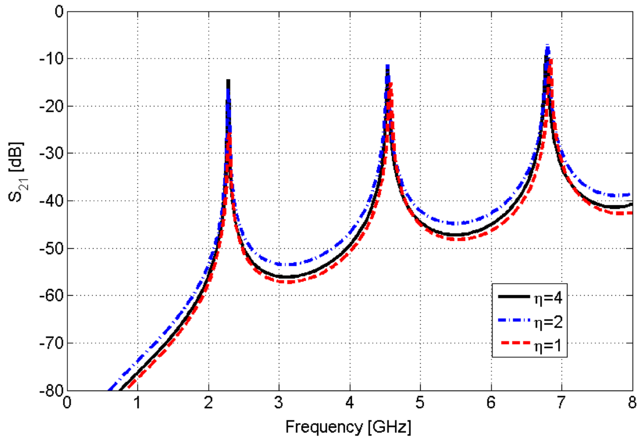


Fig. 8. Variation of the transmission factor $S_{21}$ due to different $\eta$, calculated by Sonnet for case $\nu = 1$.

physical length of the second transmission lines. They are for $\eta = 4$, as in previous case $l_2 = 11.9$ mm, for $\eta = 2$: $l_2 = 12.65$ mm, and for $\eta = 1$: $l_2 = 13.35$ mm.

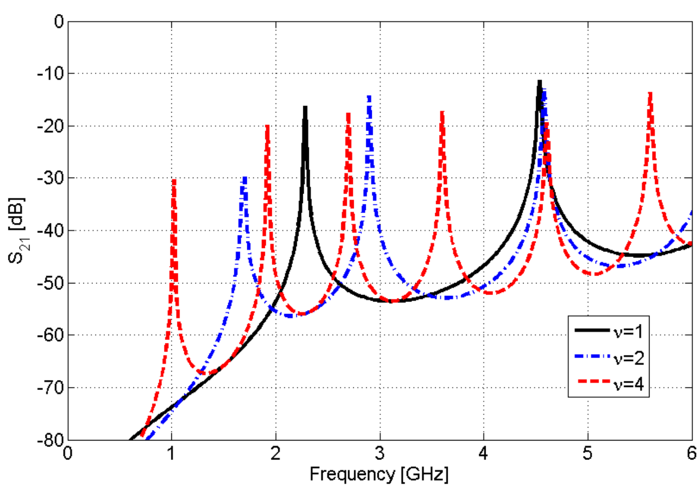


Fig. 9. Variation of the transmission factor $S_{21}$ due to different $\nu$, calculated by Sonnet for case $\eta = 2$.

The last case observed in this work is the variation of the length ratio $\nu$ with constant admittance ratio $\eta = 2$. As predicted in the previous section, increasing the segment length leads to reduction of resonant frequencies. Resonators with large length have more resonances than smaller ones. Moreover we see all resonators have the same resonant frequency at about 4.5 GHz, which is related to the phase of 3.1416.

## V. Conclusion

In this work we have theoretically analyzed a stepped impedance resonator having arbitrary admittance ratio and arbitrary length ratio. Variation of these ratios lead to appearance and moving of resonant positions. This approach was numerically verified by means of computer simulation with a commercial software. The accuracy of the predicted resonant positions is very good, i.e. about 48 MHz at 4.6 GHz and about 76 MHz at 6.8 GHz. Moreover, computer simulations verified also the other phenomenon, such unchanged resonant positions for length ratio $\mu = 1$ for all admittance ratio $\eta$.

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